

**I. PART 1. MULTIPLE CHOICES QUESTIONS (7.0 points)**

Write the correct answer (A, B, C or D) for each of the following questions in the correspondingly numbered space on your answer sheet.

**Question 1.** Quadrilateral  $ABCD$  is a rhombus with perimeter  $48\text{cm}$ . The length of the side of rhombus is equal to

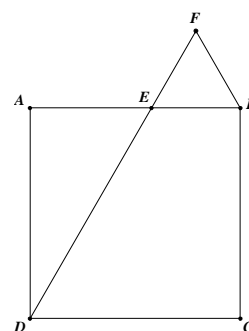
- A.  $16\text{cm}$                       B.  $9\text{cm}$                       C.  $24\text{cm}$                       D.  $12\text{cm}$

**Question 2.** Let  $ABCD$  be a parallelogram such that  $\widehat{ADB} = 90^\circ$ ,  $AB = 5$ ,  $AD = 3$ , then  $AC$  is

- A. 4                      B. 8                      C.  $2\sqrt{13}$                       D.  $\sqrt{13}$

**Question 3.** On the right figure, let  $ABCD$  be a square,  $E$  and  $F$  are two points on  $AB$  and  $DE$ , respectively such that  $BEF$  is an equilateral triangle. If  $AB = \sqrt{3}$ , then  $EF$  is equal to

- A.  $\frac{\sqrt{3}}{2}$                       B.  $2 - \sqrt{3}$   
C.  $\sqrt{3} - 1$                       D.  $\frac{\sqrt{3}}{3}$



**Question 4.** The sum of all angles in a quadrilateral is equal to

- A.  $270^\circ$                       B.  $540^\circ$                       C.  $360^\circ$                       D.  $180^\circ$

**Question 5.** Let  $Q(x) = 2x^3 + ax^2 + bx + c$  where  $a, b, c$  are given constants. If  $Q(0) = 1$ ,  $Q(-1) = 2$ ,  $Q(2) = 3$ , the value of  $a + b + c$  is equal to

- A.  $-\frac{14}{3}$                       B.  $\frac{20}{3}$                       C. 4                      D. -2

**Question 6.** Let  $a$  be a positive number such that  $\left|a - \frac{1}{a}\right| = 1$ . The value of  $a^2 + \frac{1}{a^2}$  is equal to

- A. 3                      B. -1                      C. 2                      D. 1

**Question 7.** The value of  $A = \frac{x^3 + 27}{x + 3} - \frac{x^4 - 16}{x^2 + 4}$  with  $x = \frac{1}{4}$  is equal to

- A.  $\frac{21}{5}$                       B.  $\frac{49}{4}$                       C.  $\frac{43}{4}$                       D.  $\frac{61}{4}$

**Question 8.** Given expression  $\frac{x+1}{x^2(x+2)} + \frac{3}{|x+2|}$ . The expression is well-defined with which of the following conditions of  $x$ ?

- A.  $x \neq -2$                       B.  $x \neq 0$  or  $x \neq -2$                       C.  $x \neq 0$                       D.  $x \neq 0$  and  $x \neq -2$

**Question 9.** Given trapezoid  $ABCD$  with  $AB$  and  $CD$  parallel and  $\widehat{A} - \widehat{D} = 40^\circ$ ,  $\widehat{B} = 5\widehat{C}$ . Then  $\widehat{A} + \widehat{B}$  is equal to

- A.  $100^\circ$                       B.  $140^\circ$                       C.  $220^\circ$                       D.  $260^\circ$

**Question 10.** Calculate the sum of all coefficients in the expansion of expression  $(x^2 - 2x - 3)^7$ .

- A. -4                                      B. -2187                                      C. -16384                                      D. -78125

**Question 11.** Let  $n$  be an integer. Suppose that  $n^2 + n - 2$  is divisible by  $n + 1$ . What is the value of  $n$ ?

- A. 1                                      B. 4                                      C. 2                                      D. 3

**Question 12.** What is the solution of  $A = 2x^5 - \frac{1}{2}x^3 + \frac{3}{4}x^2 - 1$  with  $x = -2$ ?

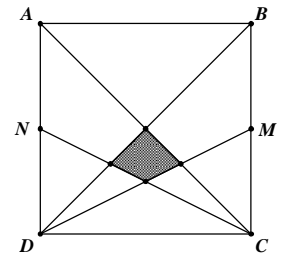
- A. -58                                      B. 64                                      C. -64                                      D. 58

**Question 13.** Factorize the polynomial  $x^4 + 2020x^2 + 2019x + 2020$ .

- A.  $(x+1)(x^2 + x + 2020)$                                       B.  $(x^2 + x + 1)(x^2 - x + 2020)$   
C.  $(x + 2020)(x^3 + 2020x + 2019)$                                       D.  $(x+1)^2(x^2 - x + 2020)$

**Question 14.** On the right figure, given square  $ABCD$  with  $AB = 10$ . Assume that  $M$  and  $N$  are mid-points of the sides  $AD$  and  $BC$ , respectively. The area of the black space is equal to

- A.  $\frac{25}{7}$                                       B. 5  
C.  $\frac{25}{6}$                                       D.  $\frac{25}{12}$



**Question 15.** Let  $ABCD$  be an isosceles trapezoid with two bases  $AB$ ,  $CD$  and  $AC$  is perpendicular to  $BD$ . The line  $AC$  meets  $BD$  at  $K$ . If  $AC = 12$  and  $AK = 4$  then  $DC$  is equal to

- A.  $4\sqrt{2}$                                       B.  $8\sqrt{2}$                                       C. 4                                      D. 8

**Question 16.** Which of the following statements is **false**?

- A.  $(2-a)[(a+1)^2 + 3] = 8 - a^3$                                       B.  $(b-a)(a^2 + b^2 + ab) = b^3 - a^3$   
C.  $(a^2 - b^2)(a^2 + b^2) = a^4 - b^4$                                       D.  $(a^2 - b^2)(a+b) = a^3 - b^3$

**Question 17.** Given trapezoid  $ABCD$  with two bases  $AB$ ,  $CD$ . Suppose that  $M$  and  $N$  are mid-points of the sides  $AD$  and  $BC$ . Let  $AC$  and  $BD$  intersect  $MN$  at  $I$  and  $K$ , respectively. If  $AB = 6\text{cm}$ ,  $CD = 8\text{cm}$  then  $IK$  is equal to

- A.  $3\text{cm}$                                       B.  $4\text{cm}$                                       C.  $7\text{cm}$                                       D.  $1\text{cm}$

**Question 18.** The quotient of the division of polynomial  $2019x^{2018}y^{2020} - 3x^4y^6$  by polynomial  $\frac{3}{2}x^3y^2$  is equal to

- A.  $\frac{6057}{2}x^{2015}y^{2018} - \frac{9}{2}xy^4$                                       B.  $1346x^{2021}y^{2022} - 2x^7y^8$   
C.  $1346x^{2015}y^{2018} - 2xy^4$                                       D.  $\frac{6057}{2}x^{2021}y^{2022} - \frac{9}{2}x^7y^8$

**Question 19.** Given square  $ABCD$ ,  $E$  is mid-point of  $AB$  and  $F$  is point on  $BC$  such that  $BF = 2FC$ . If  $AB = 6$ , then  $EF$  is

- A. 5                                      B. 6                                      C. 7                                      D. 4

**Question 20.** The common denominator of  $\frac{1}{(x-1)(4x^2-4x+1)}$  and  $\frac{x+1}{(2x-1)(x^2-2x+1)}$  is equal to

- A.  $(x-1)^2(1-2x)^2$                                       B.  $(x-1)^2(2x-1)$                                       C.  $(2x-1)(4x^2-4x+1)$                                       D.  $(2x-1)(x-1)$

**Question 21.** Find the minimum value of the expression  $2x^2 - 4|x - 2| - 8x + 3$ .

- A.  $-7$                       B.  $-6$                       C.  $3$                       D.  $-5$

**Question 22.** Given rectangular  $ABCD$  with  $AB = 4$ ,  $AD = 6$ ,  $M$  and  $N$  are two points on the sides  $AB$  and  $DC$ , respectively such that  $AM = MB$  and  $DN = 2NC$ . The area of triangle  $DMN$  is equal to

- A.  $8$                       B.  $6$                       C.  $4$                       D.  $12$

**Question 23.** The coefficient of  $x^2$  in the expansion of  $(1+x)(2+x^2)(3+x^3)\dots(7+x^7)$  is equal to

- A.  $1$                       B.  $5040$                       C.  $2520$                       D.  $28$

**Question 24.** Simplify the expression  $\left(\frac{1-x^3}{1-x} - x\right) : \left(\frac{1-x}{1+x^2}\right)$ ?

- A.  $\frac{(1+x^2)^2}{1-x}$                       B.  $1-x$                       C.  $\frac{1+x^2}{1-x}$                       D.  $\frac{x^2+x+1}{1+x^2}$

**Question 25.** Simplify the expression  $\frac{1}{x^2-3x+2} - \frac{3}{4-2x}$ ?

- A.  $\frac{3x-1}{x^2-3x+2}$                       B.  $\frac{1-3x}{2(x-1)(x-2)}$                       C.  $\frac{5-3x}{2(x^2-3x+2)}$                       D.  $\frac{3x-1}{2(x-1)(x-2)}$

**Question 26.** Suppose that  $x$  and  $y$  are two integer numbers such that  $x - y - xy = 79$  and  $x^2 + y^2 = 130$ .

The value of  $(x - y + 1)^2$  is equal to

- A.  $209$                       B.  $288$                       C.  $260$                       D.  $289$

**Question 27.** The value of expression  $A = x^5 - 3x^2y^3 + 3x^3y^2 - y^5$  with  $x = -2$  and  $y = 3$  is equal to

- A.  $-103$                       B.  $-167$                       C.  $-815$                       D.  $-383$

**Question 28.** Given triangle  $ABC$  with the right angle  $A$ . Let  $M$  be a point on  $BC$  such that  $BM = 2MC$ . If  $AB = 3\text{cm}$ ,  $AC = 4\text{cm}$  then the area of triangle  $ACM$  is equal to

- A.  $2\text{cm}^2$                       B.  $6\text{cm}^2$                       C.  $4\text{cm}^2$                       D.  $3\text{cm}^2$

**Question 29.** A square is inscribed inside a rhombus with diagonals  $6$  and  $10$ . Find the area of the square.

- A.  $\frac{225}{64}$                       B.  $\frac{15}{4}$                       C.  $\frac{225}{16}$                       D.  $\frac{15}{8}$

**Question 30.** Given the lengths of sides of a rectangular are  $6\text{cm}$  and  $8\text{cm}$ , respectively. What is the length of the diagonal?

- A.  $8\text{cm}$                       B.  $6\text{cm}$                       C.  $10\text{cm}$                       D.  $12\text{cm}$

**Question 31.** What is the value of  $x$  in  $\frac{-3x^2 + 2x + 1}{x^2 + 3x - 4} = 1$ ?

- A.  $x = -\frac{1}{3}$                       B. Does not exist  $x$   
C.  $x = -\frac{5}{4}$                       D.  $x = 1$  or  $x = -\frac{1}{3}$

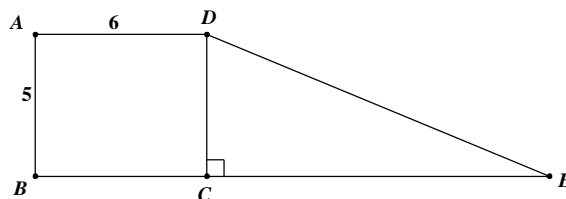
**Question 32.** Given trapezoid  $ABCD$  with two bases  $AB = 3\text{cm}$ ,  $CD = 13\text{cm}$ . Suppose that the area of the trapezoid is  $48\text{cm}^2$ . The height  $AH$  is equal to

- A.  $6\text{cm}$                       B.  $3\text{cm}$                       C.  $24\text{cm}$                       D.  $1,5\text{cm}$

**Question 33.** Let  $a, b$  be a real numbers such that  $\frac{2x+1}{x^2+6x+9} = \frac{a}{x+3} + \frac{b}{(x+3)^2}$ . The value of  $a+b$  is equal to

A. 1                                      B. -3                                      C. -5                                      D. 7

**Question 34.** On the right figure, given rectangle  $ABCD$  and right triangle  $DCE$  which have the same area. The height  $DE$  is equal to



- A. 13                                      B.  $5\sqrt{2}$   
C. 12                                      D.  $\sqrt{125}$

**Question 35.** Suppose that polynomial  $x^4 + 2x^3 + x^2 + m$  is divisible by polynomial  $x^2 + x - 1$ . The value of  $m$  is

A. 1                                      B. 4                                      C. -2                                      D. -1

## II. PART II. PROBLEM SOLVING (3.0 points)

*Write the solutions to the following problems in the provided space on your answer sheet.*

**Problem 1.** Find all prime numbers  $p$  such that  $16p+1$  is a perfect cube.

**Problem 2.** A pair of numbers are *twin primes* if they differ by two and both are primes. Prove that, except the pair  $\{3;5\}$ , the sum of any pair of twin primes is a multiple of 12.

**Problem 3.** In rectangle  $ABCD$ , the length of side  $AB$  is twice as the length of side  $BC$ . A point  $P$  is taken on side  $AB$  such that  $BP = \frac{1}{4} AB$ . Show that  $BD$  is perpendicular to  $CP$ .

**-THE END-**

Student's full name: ..... Student's ID: .....

First observer's name and signature: ..... Second observer's name and signature: .....